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Research paper

Modeling diameter distribution of Aleppo pine (*Pinus halepensis* Mill.) natural forest in the Aures (Algeria) using the Weibull, Beta and Normal distributions with parameters depending on stand variables

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ABSTRACT 230 temporary plots located in Aleppo pine (*Pinus halepensis* Mill.) stands in the Aures (Algeria) were used for modeling its structure with three theoretical distributions, i.e., the Weibull; the normal and the Beta one. Parameters of the Weibull distribution were estimated using two methods: the maximum likelihood and the method of moments. Diameter distribution models were obtained by estimation of each distribution parameters and by their prediction using stand variables. Results revealed the efficiency of the Weibull distribution estimated with the method of moments. The parameter estimation method is more accurate compared to parameter recovery method despite the existence of strong correlations between parameters of the theoretical distributions and some population variables such as arithmetic or quadratic mean diameter and dominant height. Plot characteristics revealed the existence of several distribution shapes: symmetrical; dissymmetrical with left asymmetry and reverse *I* or *J*-shaped distributions.

KEYWORDS: *Pinus halepensis*, Algeria, stem distribution, diameter class, distribution modeling, stand variables.

Introduction

Aleppo pine (*Pinus halepensis* Mill.) has a circum-mediterranean range extending mainly from Greece to the Maghreb (Nahal 1962). In Algeria, it represents the first forest species in terms of area with 881,000 ha corresponding to 21% of the principal forest types (Djema and Messaoudène 2009). The species is found in the majority of Mediterranean bioclimatic variants of Algeria and composes important forests with appreciable ecological values even in the most hostile areas at the limit of the Sahara (Quézel 1986). It is found both in coastal and inland areas. Its plasticity has been reported for various aspects including warming-induced drought stress (Choury et al. 2017 and references therein).

In the Aures region, which is a part of the Saharan Atlas mountains, Aleppo pine forest stands of Beni-Imloul produce an annual volume of 40,000 m^3 and those of Ouled-Yakoub and Beni-Oudjana, whose combined area amounts to more than 40,000 hectares, may potentially produce more than 8,500 m^3 annually (BNEF 1984). The productivity of Ouled-Yakoub Aleppo pine forest, at an age of 70 years, averages 2 m^3 ha^{-1} $year^{-1}$ (range: 0.5 to 4.8 m^3 ha^{-1} $year^{-1}$). The maximum productivity is reached around 50 years. The target diameter for timber production is generally greater than 35 cm and such diameters are obtained between 70 and 90 years of age depending on site fertility. The standing timber volume before the final harvest usually ranges from 153 to 172 m^3 ha^{-1}. For a site of a medium fertility, a diameter of 35 cm corresponds to an exploitability age of 70 years (Bentouati and Bariteau 2005).

Recent studies indicate the potential role for the species as a source of bioresources such as bark tannins-based adhesives (Saad et al. 2014); herbicidal properties of leaf, stem and cone essential oils (Amri et al. 2013); seed-based nutrient additives in food industry (Kadri et al. 2015) and timber biobased materials for building (Liman et al. 2016). These examples indicate that besides solid wood, harvest residues (branches, leaves and cones) could also be valued for various uses unless otherwise recommended by conservation considerations such as conservation of biotic assemblages (e.g. Dahlberg et al. 2011).

A sustainable timber production should rely on sound silvicultural and management practices of Aleppo pine forests. Few studies (Bentouati 2005, Bentouati and Bariteau 2005) have been devoted to silviculture of Aleppo pine in the Aures region. These preliminary studies mainly concerned productivity. They proposed a silvicultural model based on the actual state of the stands. Another, more dynamic silvicultural model, based

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on the management of competition between trees, is being prepared (work not yet published). A sustainable management requires knowledge on stem distribution by category of size and, more importantly, establishing a model which could be used in yield tables (Vanclay 1994). A frequency distribution of stems by category of size is of great importance in forest management because it reflects stand structure and its response to growing conditions and to silvicultural treatments. In an uneven-aged stand, stem dimensions vary greatly, hence the need to cluster them into classes of equal range similar to a distribution curve. According to Rondeux (1993), a mathematical model that fits the distribution of the number of stems by classes of diameter addresses can be used to address several types of concerns: *(i)* construction and use of management tools such as production or growth models by size classes; *(ii)* study of stand structure; *(iii)* estimation of the quantity of woody material by assortment (i.e. type of product) or by size category; *(iv)* simulation of silvicultural treatment standards.

Many theoretical distributions have been used to model the structure of forest stands. Several studies aimed at comparing the precision of several theoretical probability ditributions in order to obtain the best fit of the structure with the theoretical distribution. In this context, it is worth mentioning the use of the Jedlinski deciles and the lognormal distribution (Dagnelie and Rondeux 1971) and the Pearson system for unimodal distributions (Sghaier and Palm 2002, Fonton and Sagbo 2004, Sghaier and Ammari 2012). Several estimations methods aimed at optimizing the accuracy of estimators of the distribution parameters. Liu et al. (2004) compared the effectiveness of three estimators: the maximum likelihood method, the method of moments and the percentile method. A statistical method based on nonlinear regression was developed by Abd kudus (1999) and linear regression method was also used and compared with percentiles by Hudak and Tiryakioglu (2009).

Comparison of the Weibull distribution accuracy with that of the normal distribution (Lejeune 1994, Sghaier et al. 2016); the Beta distribution (Maltamo et al. 1995) and Johnson's system (Zhoo and Mc Tague 1996, Zhang et al. 2003) indicated that among all the above mentioned distributions, the Weibull distribution better fit the diameter distribution in general. Such a distribution is characterized by a great flexibility of use (Rabhi et al. 2016) and is quite commonly used in specialized forest literature (Bailey and Dell 1973, Gorgoso et al. 2007, Lei 2008) due to its great flexibility and the existence of an explicit form of its distribution function, on one hand, and its ability to describe a wide range of uni-modal distributions, including the frequency - inverted one, on the other hand. The truncated Weibull distribution was used to model the basal area diameter distribution of *P. sylvestris*, *P. nigra* and *P. halepensis* stands in Catalonia (Spain) (Palahi et al. 2006). Authors show that although the stand structures varied widely, the Weibull function performed well in most of the cases.

The studied species has not been the subject of a distribution model. Given its importance, the purpose of this study was to compare the accuracy of the normal, beta and Weibull distributions for describing diameter distributions in even-aged stands of *Pinus halepensis* in the regions of Aures (NE Algeria). The maximum likelihood and moments methods were used and compared to estimate the two parameters of Weibull distribution. Two different approaches, namely, parameters estimation and parameters prediction with stand variables or parameter recovery, were used to estimate the distributions' parameter.

Material and methods

Study sites and field data

The pine forest of Ouled-Yakoub, located in the great massif of Aures (NE Algeria) has not been managed. We report that strong thinning from above was practiced during the 1970s. This choice was justified mainly, at that time, by the advanced age of most of the pine forest and the health of the trees. Except sporadic cuts (firewood and construction for household needs), the lack of regular logging affecting the entire massif following a management plan resulting in irregular and complex structures.

A pure random design (Duplat and Perrotte 1981) was established in this pine forest in which 230 circular plots of 10 ares were installed by reading the table of random numbers. The selected plots are located in pure stands, normally dense and without gaps. The inventory of the plots concerned measurements of diameter and circumference at 1.3m and of the total height of all the trees with diameter at 1.3m ≥ 7 cm. A Pressler increment borer were used to extract increment cores. Age was estimated from average basal area tree of each plot (assumed even-aged) by counting the number of rings at 0.50 m.

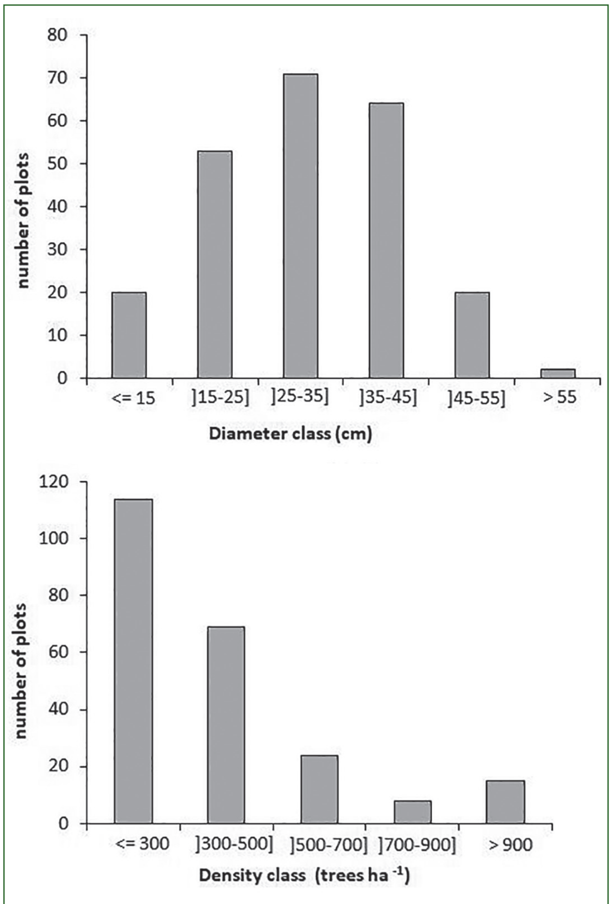
The geological substratum dates back to the upper Cretaceous (Lafitte 1939). The bioclimate corresponds to fresh and cold semi-arid. The vegetation cover comprises Aleppo pine as an upperstorey species, accompanied by trees and shrubs such *Quercus rotundifolia* L. and *Junipersus oxycedrus* L., etc. in the understorey and a more or less abundant herbaceous layer depending on slope and soil erosion. Rendzina soil types are present in these forests (Bentouati 2005) and soil depth is below 20 cm.

Adjustment of a probability distribution requires a large number of measurements. As whole, 6,394 stems were recorded across 230 plots. The main dendrometric characteristics of the sampled plots are shown in Table 1. The distribution of all plots by category of size revealed a bell shaped distribution (Fig.1). The extreme classes (i.e. low and large diameters) are poorly represented with values above 55 cm representing less than 2% of the sampled trees. Most trees had a diameter ranging between 16 and 45 cm. Distribution of plots by category of density (Fig.1) indicates the highest frequency of low-density plots and the low frequency of plots with densities exceeding 700 stems ha^{-1}, resulting in the low average density (i.e. 385 trees ha^{-1}) of the 230 sampled plots.

Table 1 - Statistical parameters of measured stand variables (number of observations = 230).

Stand variables	Min	Max	Mean	Standard deviation
Age (years)	26	124	67.72	22.27
Dominant height «Hdom» (m)	7.67	23.7	14.18	3.35
Mean height «Hm» (m)	6	20.66	12.51	3.2
Quadratic mean diameter «dg» (cm)	9.87	68.46	33.35	11.96
Mean diameter « DBH » (cm)	9.86	62.94	30.89	10.85
Density « N » (trees ha -1)	160	1,760	385	247.62

Figure 1 - Distribution of the 230 plots per class of mean diameter and mean density.



Studied distributions

The normal distribution

The probability density function (PDF) of the normal distribution can be expressed as follows (1) (Dagnelie 2013):

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right] \qquad \text{(eq. 1)}$$

where x is the random variable, and m and σ are its arithmetic mean and standard deviation, respectively.

Estimation of the mean parameter m and standard deviation σ was done with the following relationships (2 and 3):

$$\hat{m} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \text{(eq. 2)}$$

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \hat{m})^2} \qquad \text{(eq. 3)}$$

where n indicates the number of trees per plot and x_i (cm) the diameter at breast height of each tree.

The Beta distribution

The probability density function (PDF) of the Beta distribution has the following shape (4) (Gorgoso et al. 2012):

$$\begin{cases} f(x) = c(x-L)^{\alpha}(U-x)^{\lambda} & if \quad L \le x \le U \\ f(x) = 0 & if\ not \end{cases} \qquad \text{(eq. 4)}$$

With:

x: the random variable (diameter at 1.30 m).

c: the function scale factor which ensures the equality (5):

$$c\int_{L}^{U} c(x-L)^{\alpha}(U-x)^{\lambda}dx = 1 \qquad \text{(eq. 5)}$$

L and U: The lower and upper limits of the distribution.

α and λ: The shape parameters of the distribution.

The lower limit L of the distribution may take values such as 0, min (the minimum observed diameter in the plot), min/2, etc. The upper limit of the distribution may take a higher or a lower value than the maximum observed diameter in the plot ($U \ge d_{max}$).

Estimation of parameters (6, 7 and 8) was achieved with the method of moments which is the only estimation method used in forestry (Gorgoso et al. 2012).

$$\lambda = \frac{\frac{z}{s_{rel}^2(z+1)^2} - 1}{z+1} - 1 \qquad \text{(eq. 6)}$$

$$\alpha = z(\lambda + 1) - 1 \qquad \text{(eq. 7)}$$

Where:

$$z = \frac{x_{rel}}{1-x_{rel}}; \quad x_{rel} = \frac{\bar{d}-L}{U-L} \quad \text{and} \quad s_{rel}^2 = \frac{s^2}{(U-L)^2}$$

$$c = \frac{1}{c\int_L^U c(x-L)^{\alpha}(U-x)^{\lambda}dx} = \frac{(U-L)^{-\alpha}\Gamma(2+\alpha+\lambda)}{(U-L)^{1+\lambda}\Gamma(1+\alpha)\Gamma(1+\lambda)} \quad \text{(eq. 8)}$$

s^2 et $\bar{d}$: sample variance and mean, respectively.
Γ : Gamma function.

The values used for the L and U limits of the beta distribution, in this study, are 0 and (d_{max} + 1). The value 1 is added to d_{max} (maximum diameter) as the upper limit (U) to ensure a non-zero estimate of maximum tree diameters in the plot.

The two parameter - Weibull distribution

In some studies, the parameter a is arbitrarily fixed at 0.5 d_{min} (Lei 2008) or at zero (Gorgoso et al. 2007), thus reducing the function to a two-parameter Weibull distribution which is easier to model and provides similar results to those of the three-parameter Weibull function.

The probability density function of the two parameter-Weibull distribution has the following shape (9) (Gorgoso et al. 2012, Sghaier et al. 2016):

$$f(x) = \frac{c}{b}\left(\frac{x}{b}\right)^{c-1} exp\left[-\left(\frac{x}{b}\right)^{c}\right] \quad \text{(eq. 9)}$$

With:
b: scale parameter.
c: shape parameter.

Two methods were used to estimate the parameters of the Weibull distribution, namely the maximum likelihood method (ML) and the method of moments (MoM).

The Maximum Likelihood Method

The maximum likelihood method is a commonly used procedure for the Weibull distribution in forestry because it has certain desirable properties (Lei 2008). Estimation of the parameters using maximum likelihood has been found to produce consistently better goodness-of-fit statistics compared to other methods, but it also puts the greatest demands on the computational resources (Cao and McCarty 2006). If we consider the Weibull probability density function, then the likelihood function (L) will be [10]:

$$L(x_1, \dots, x_n; b, c) = \prod_{i=1}^{n} \frac{c}{b}\left(\frac{x_i}{b}\right)^{c-1} exp\left[-\left(\frac{x_i}{b}\right)^{c}\right] \quad \text{(eq. 10)}$$

Taking the logarithms from this equation, differentiating with respect to b and c respectively, and satisfying the following equations (11 and 12) (Nanos and Montero 2002, Eerikainen and Maltamo 2003):

$$\hat{c} = \left[\frac{\sum_{i=1}^{n} x_i^c \times log\,(x_i)}{\sum_{i=1}^{n} x_i^c} - \frac{1}{n}\sum_{i=1}^{n} log\,(x_i)\right]^{-1} \quad \text{(eq. 11)}$$

$$\hat{b} = \left[\frac{1}{n}\sum_{i=1}^{n} x_i^c\right]^{\frac{1}{c}} \quad \text{(eq. 12)}$$

where n equals the number of sample observations in a Weibull distribution and x_i the diameter of each tree. The value of c must be obtained by using standard iterative procedures and then it is used obtain b.

The method of moments

The method of moments is another technique commonly used for parameter estimation. In the Weibull distribution, the k moment readily follows from the probability density function (13) (Lei 2008):

$$m_k = \left(\frac{1}{b}\right)^{k/c} \Gamma\left(1 + \frac{k}{c}\right) \quad \text{(eq. 13)}$$

Then this equation, we can find the first and the second moment as follows:

$$m_1 = \mu = \left(\frac{1}{b}\right)^{1/c} \Gamma\left(1 + \frac{1}{c}\right) \quad \text{(eq. 14)}$$

$$m_2 = \mu^2 + \sigma^2 = \left(\frac{1}{b}\right)^{2/c} \Gamma\left(1 + \frac{2}{c}\right) \quad \text{(eq. 15)}$$

which gives:

$$\sigma^2 = m_2 - \mu^2 = \left(\frac{1}{b}\right)^{2/c} \left[\Gamma\left(1 + \frac{2}{c}\right) - \Gamma^2\left(1 + \frac{1}{c}\right)\right] \quad \text{(eq. 16)}$$

where σ^2 is the variance of tree diameters in a plot, and m_1 (14), m_2 (15) are the arithmetic and quadratic mean diameter in a plot, respectively. When σ^2 (16) is divided by the square of m_1, the expression for obtaining c (17) is :

$$\frac{\sigma^2}{\mu^2} = \frac{\left(\Gamma(1+\frac{2}{c}) - \Gamma^2(1+\frac{1}{c})\right)}{\Gamma^2(1+\frac{1}{c})} \Longrightarrow$$

$$\sigma^2 = \frac{\bar{d}^2}{\Gamma^2(1+\frac{1}{c})}\left(\Gamma^2\left(1 + \frac{2}{c}\right) - \Gamma^2\left(1 + \frac{1}{c}\right)\right) \quad \text{(eq. 17)}$$

In order to estimate b and c, we need to calculate the arithmetic mean diameter $\bar{d}$ and the variance σ^2 of the observed distribution and obtain the estimator of c. Last equation was resolved by an iterative procedure. When the value of the location parameter (a) is zero, the scale parameter (b) can then be calculated directly using the following equation (18) (Gorgoso et al. 2007):

$$b = \frac{\bar{d}}{\Gamma(1+\frac{1}{c})} \quad \text{(eq. 18)}$$

where $\bar{d}$ is the arithmetic mean diameter.

Prediction of parameters of the distributions as a function of stand variables

To make the stem distribution models dynamic and dependent on stand characteristics, correlations between parameter estimates of the studied distributions and stand characteristics were calculated.

Linear and nonlinear regressions were implemented in order to obtain models explaining parameters of such distributions according to stand descriptors. Unlike the first approach, which consists in estimating the parameters of each distribution directly from the raw data (i.e. diameter at 1,30 m) and which is called the parameter estimation method, the second approach (the parameter recovery method), aims at predicting the same parameters from stand characteristics (Vanclay 1994). With this second approach, stem distribution models can be used to partition trees within-stand by diameter class, either directly, from the average magnitudes obtained from the plots of measurement, or indirectly, as complementary tools to yield tables.

The parameter recovery approach may offer a more robust alternative. The parameters of the distribution are predicted indirectly by matching the moments of the distribution to predicted stand attributes such as stand basal area and mean diameter. This approach is an efficient way to estimate the parameters of the Weibull distribution (e.g. Reynolds et al. 1988 in Vanclay 1994). The parameter prediction models developed by Palahi et al. (2006) enable one to predict the basal area diameter distribution for a given stand of *P. sylvestris*, *P. nigra* or *P. halepensis* using rather limited information (stand basal area, number of trees per hectare or quadratic mean and elevation). Sghaier et al. (2016) note that the highest Pearson correlation between the parameters of Normal and Weibull distribution and the stand variables of *Tetraclinis articulata* stands was obtained with quadratic mean and its natural logarithm transformations. Stand characteristics we tested in the models were: stand age (Age), density (N), dominant height (H_d), average height (H_m), quadratic mean diameter (d_g), arithmetic mean diameter ($\bar{d}$), the first percentile (P_{25}); the median (P_{50}); the third percentile (P_{75}); the quotient of the first percentile; the median and the third percentile on the quadratic mean diameter (respectively RP_{25}, RP_{50} and RP_{75}). Sghaier et al. (2016) took also into account the logarithmic transformation of the quotient of the first percentile; the median and the third percentile (respectively LP_{25}, LP_{50}, LP_{75}) on the quadratic mean diameter.

Simulation of the proportion of trees by class of diameter

For each studied distribution and each estimation method, the proportion of trees pertaining to a class of diameter whose limits would be l_1 and l_2, equals the integral of the probability density function on such interval, which means the following:

$$\int_{l_1}^{l_2} f(x)dx$$

Such proportion can be represented by the area beneath the curve: y = f(x) delineated by two perpendicular lines to the abscissa axis, elevated to the inferior and superior limits of the class.

To determine the number of trees per class, the area beneath the density curve was divided, by vertical rows, into *n* parts of constant area which equals *1/n* with *n* representing the number of trees in a plot. Each elementary area corresponds to a given tree. The continuous theoretical distribution of diameters was then replaced by a custered distribution with a constant class interval of 1 cm. This allows estimation of the total number of trees for each diameter class.

Comparison criteria

The goodness of fit test can be used at different steps of the distribution modeling. During estimation of parameters, it is necessary to test the concordance between the theoretical and the observed distributions, in order to have an idea on the capacity of the chosen distribution to represent the type of stand concerned by the model.

Use of conventional "Goodness-of-fit" tests poses some problems. The Kolmogorov and Smirnov (K-S) test seeks the highest distance between empirical and cumulated distribution functions. This test applies only in the case of continuous data and its use for discrete data is valid only with a modified version (Lafond 2010).

Despite its drawback, use of the K-S test showed significant efficiency in comparison of six adjustment methods by Liu et al. (2009) and a perfect concordance with the error index implemented (i.e. Reynolds error index).

The Chi-square Pearson test compares the predicted and observed numbers for each class of diameter. It has the advantage of suitability to discrete data but it is sensitive to class definition, which has a strong influence on the test results. Its use often requires pooling of extreme classes in the case of insufficient observations (Dagnelie 1973). Similarly, the Chi-square test has the disadvantage of testing equality of distributions what is probably much strict in the case of models' evaluation (Lafond 2010). Therefore, it would be more practical to use Chi-square value as a measure of distance between distributions.

Finally, three numerical criteria and graphs of residuals (difference between observed and estimated numbers of trees per class of diameter) were used (19, 20 and 21):

$$Bias = \frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right) \quad \text{(eq. 19)}$$

$$Mean\ absolute\ error\ (MAE) = \frac{1}{n}\sum_{i=1}^{n}\left|Y_i - \hat{Y}_i\right| \quad \text{(eq. 20)}$$

$$Root\ mean\ square\ error\ (RMSE) = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n-p}} \quad \text{(eq. 21)}$$

Where Y_i and $\hat{Y}_i$ represent respectively the observed and the estimated numbers of stems per class of diameter and per plot; $\bar{Y}$ the mean number of stems par class of diameter for all plots; p the number of parameters in the equation of the studied distribution and n the total number of classes of diameter for all plots.

Reynolds index in %

Reynolds index (22) (Reynolds et al. 1988) consists in summing, for the k classes of diameter of each plot, the absolute difference between predicted and observed numbers:

$$EI = \sum_{i=1}^{k} \left| Y_i - \hat{Y}_i \right| \qquad \text{(eq. 22)}$$

In such index, we can multiply the absolute difference between the observed and the estimated number for each class of diameter by the corresponding volume, using a one entry volume table (23) built with 340 cubed stems of the same plots (Bentouati 2005).

$$v = 42.093 - 7.323d + 0.612d^2 \qquad \text{(eq. 23)}$$

with $R^2_{aj} = 0.9441$ and residual standard deviation ($\hat{\sigma}_r = 109.782\ dm^3$).

Where the volume v is expressed in dm^3 and the diameter d corresponds to the centre of diameter class interval. That way, Reynold's index will take the following shape (24):

$$EI = \sum_{i=1}^{k} \left(\left| Y_i - \hat{Y}_i \right| \times v_i \right) \qquad \text{(eq. 24)}$$

The same index can be expressed in percent of plot total volume (Lejeune 1994, Sghaier et al. 2016) as following (25):

$$EI' = \frac{EI}{\sum_{i=1}^{k} Y_i \times v_i} \times 100 \qquad \text{(eq. 25)}$$

The later index (EI') expresses the percentage of error on the volume for each distribution and for each of the 230 studied plots.

Normality of residuals

Normality of error distribution was examined by the Ryan-Joiner normality test (Ryan and Joiner 1976) and the plot of normal quantiles (QQ-probability plots). The Ryan-Joiner test is a simple alternative to the Shapiro-Wilk normality test used most often in statistical softwares. The principle of this test is based on calculation of the correlation coefficient between residuals ($e_i = Y_i - \hat{Y}_i$), ranked in ascending order, and normal scores or normal quantiles (zi), with s_2 as the variance of these residuals (26):

$$\rho_{obs} = \frac{\sum_{i=1}^{n} e_i z_i}{\sqrt{s^2 (n-1) \sum_{i=1}^{n} z_i^2}} \qquad \text{(eq. 26)}$$

Normal scores and normal quantiles (z_i) were calculated as following (27):

$$z_i = \Phi^{-1}\left(\frac{i-3/8}{n+1/4}\right) \qquad \text{(eq. 27)}$$

Where $\Phi^{-1}(z)$ is the inverse of the cumulative distribution function of the reduced normal distribution.

A value ρ_{obs} close to 1 indicates that the distribution of residuals can be considered as normal. Normality of residuals must be rejected at a confidence level of $(1-\alpha)$ when $\hat{\rho}_{obs} < \rho_\alpha$. Values of ρ_α are read on a table as a function of the number of observations n (Looney and Gulledge 1985). By providing an idea on the linearity of the relationship, the plot of the zi and ei constitutes a first step of this test.

Results

Parameter estimation

It appears from Table 2 that the two - Weibull distribution estimation methods provided similar results. Regarding deviation, evaluated by the coefficient of variation, it was low and did not exceed 37% for the parameters of the normal and Weibull distributions, while such coefficient was high for the Beta distribution parameters, particularly the

Table 2 - Descriptive statistics for the estimated parameters of the three studied distributions.

Distribution	Method	Parameter	Mean	Minimum	Maximum	CV%
Weibull	ML	b	34.348	10.898	67.243	34.46
		c	3.526	1.470	7.606	32.77
	MoM	b	34.318	10.872	67.552	34.58
		c	3.487	1.377	8.215	33.77
Beta	-	c	0.000089	1.36x10-24	0.0047	527.81
		α	2.516	-0.278	10.668	66.14
		λ	1.529	-0.306	6.353	72.46
Normal	-	$\hat{m}$	30.90	9.87	62.94	35.14
		$\hat{\sigma}$	10.422	2.638	19.374	36.14

ML: the maximum likelihood method. MoM: the method of moments.

scale parameter *c* which displayed a coefficient of variation of about 528 %. This result may be motivated by extreme low values of magnitude. The great variability observed comes from the fact that the latter is calculated as a function of the other two parameters α and λ of the Beta distribution using the equation (8).

Parameter recovery

Table 3 provides the Pearson correlation values between the estimated parameters of the different distributions and stand characteristics, used as explanatory variables (see methodology).

The calculated correlations showed a perfect agreement between the two estimation methods of Weibull distribution regarding the degree of the relationship with stand variables. However, the two parameters *b* and *c* did not show the same result with respect to the same stand characteristics since all the correlations with the parameter *b* were significant while in the case of the parameter *c*, an absence of correlation was noticed with density; LP_{75} and average and dominant heights. The best correlations between parameters of the studied distributions and stand characteristics, concern the average diameter ($\bar{d}$) for the parameter *b* and the first percentile (P_{25}) for the parameter *c* of the Weibull distribution according to the two estimation methods; the third percentile (P_{75}) for the standard deviation ($\hat{\sigma}$) of the normal distribution and the parameter λ of the beta distribution, and finally the logarithm of the ratio of P_{25} to the quadratic mean diameter (d_g) for the parameter λ of the same distribution.

Table 3 - Correlation between the estimated parameters and the means of plot characteristics.

	Normal	Weibull (ML)		Weibull (MoM)		Beta		
Variable	$\hat{\sigma}$	b	c	b	c	c	α	λ
Age	0.470***	0.691***	0.221***	0.691***	0.209**	0.012	0.117	-0.190**
N	-0.551***	-0.683***	-0.103	-0.682***	-0.056	0.020	0.118	0.446***
H_d	0.574***	0.678***	0.077	0.677***	0.053	0.143*	-0.062	-0.322***
H_m	0.558***	0.662***	0.097	0.662***	0.066	0.154*	-0.061	-0.365***
dg	0.556***	0.838***	0.285***	0.838***	0.249***	-0.023	0.085	-0.361***
$\bar{d}$	0.577***	**0.988*****	0.423***	**0.988*****	0.377***	-0.065	0.179**	-0.384***
P_{25}	0.325***	0.932***	**0.630*****	0.934***	**0.594*****	-0.203**	0.411***	-0.221***
P_{50}	0.524***	0.978***	0.472***	0.979***	0.421***	-0.112	0.215***	-0.372***
P_{75}	**0.711*****	0.978***	0.276***	0.977***	0.223***	0.041	0.015	**-0.492*****
RP_{25}	-0.175**	0.337***	0.545***	0.340***	0.536***	-0.303***	0.463***	0.090
RP_{50}	0.034	0.279***	0.265***	0.280***	0.241***	-0.161*	0.160*	-0.083
RP_{75}	0.240***	0.189**	-0.046	0.189**	-0.071	0.078	-0.136*	-0.216***
LP_{25}	-0.242***	0.322***	0.584***	0.326***	0.578***	**-0.398*****	**0.514*****	0.150*
LP_{50}	-0.005	0.295***	0.319***	0.297***	0.294***	-0.204**	0.209**	-0.060
LP_{75}	0.246***	0.198**	-0.041	0.197**	-0.069	0.100	-0.145*	-0.239***
Ld	0.035	0.222***	0.200**	0.223***	0.183**	-0.089	0.123	-0.054

**** Significant: p<0.001; ** Significant: p<0.01; * Significant: p<0.05; Age (years); N: density (trees ha^{-1}); Hd: dominant height (m); H_m: mean height (m); d_g: quadratic mean diameter (cm); $\bar{d}$: arithmetic mean diameter (cm); P_{25}: 25% percentiles (cm); P_{50}: 50% percentiles (cm); P_{75}: 75% percentiles (cm); RP_{25}: (P_{25}/d_g); RP_{50}: (P_{50}/d_g); RP_{75}: (P_{75}/d_g); LP_{25}: ln (P_{25}/d_g); LP_{50}: ln (P_{50}/d_g); LP_{75}: ln (P_{75}/d_g); Ld = ln($\bar{d}/d_g$); ln: Neperian logarithm.*

Table 4 shows values, significance and accuracy of the parameters of the different models which link the estimated parameters, of the studied distributions, to the stand characteristics. For the Beta distribution, only the two parameters α and λ were related to plot characteristics. The parameter *c* is a scale parameter that must ensure that the area under the curve of the probability density function equals the unit. The calculation of this parameter, function of the values of the two parameters α and λ, being carried out by program by calling gamma function (8).

The regressions presented in the Table 4 were fitted using the Ordinary Least Squares (OLS) method. Each equation was fitted on its own and independently of the other equations. Only the variables which show the highest correlations with the dependent variables of the different functions tested (Tab. 3) were used as independent variables. The regressions presented in the Table 4 were fitted using the Ordinary Least Squares (OLS) method. Each equation was fitted on its own and independently of the other equations. Only the variables which show the highest correlations with the dependent variables of the different functions tested (Tab. 3) were used as independent variables.

No variable selection method was used given the small number of independent variables retained for the adjustment (the only variables showing strong correlations with the dependent variables: parameters of the rod distribution functions tested). For each equation, different linear and non-linear relationships were fitted and compared. The selected equations are those in which all the regression coefficients are significant at 5% error, with a maximum R^2 and a minimum residual standard deviation (RMSE).

Since the two percentiles (P_{25} and P_{75}) were used as explanatory variables to predict some parameters of the studied distributions, a relationship between such distributions and the mean diameter of the stand was fitted for each of them (Tab. 4). On the other hand, since the mean diameter of the stand has also been selected as an explanatory variable for predicting some parameters and in order to allow use of these distributions for yield tables, which generally provide only the mean square diameter, a relationship between the mean diameter and the mean square diameter was established (Tab. 4).

Table 4 - Relationship between the estimated parameters of the three studied distributions and plot characteristics.

Distrib.	Method	Equations	Parameters a_1	a_2	a_3	R^2	$\hat{\sigma}_r$
Weibull	ML	$b = a_1 + a_2\bar{d}$	1.080**	1.077***	-	0.975	1.862
		$c = a_1 + a_2 P_{25}^2$	2.622***	0.00138***	-	0.447	0.861
	MoM	$b = a_1 + a_2\bar{d}$	0.958**	1.080***	-	0.976	1.854
		$c = a_1 + a_2 P_{25}^2$	2.609***	0.00134***	-	0.406	0.910
Beta	MoM	$\alpha = a_1 + a_2(P_{25}/d_g) + a_3(P_{25}/d_g)^2$	-4.147***	14.273***	-6.387***	0.320	1.379
		$\lambda = a_1 P_{75}^{a_2}$	36.311***	-0.913***	-	0.338	0.904
Normal	-	$\bar{d} = \bar{d}$	-	-	-	-	-
		$\hat{\sigma} = a_1 P_{75}^{a_2}$	0.859***	0.692***	-	0.526	2.599
P_{25}		$P_{25} = a_1 + a_2\bar{d}$	-4.044***	0.889***	-	0.879	3.581
P_{75}		$P_{75} = a_1 + a_2\bar{d}$	2.463***	1.140***	-	0.924	3.569
$\bar{d}$		$\bar{d} = d_g - e^{(a_1 H_d)}$	0.0694***	-	-	0.663	6.303

**** Significant: p<0.001; ** Significant: p<0.01; R^2: coefficient of determination; $\hat{\sigma}_r$: residual standard deviation; ln : neperian logarithm.*

Comparison of the studied distributions

Table 5 shows results related to various comparison criteria of the studied distributions for the two methods of parameter calculation, i.e. the parameter estimation method and the parameter recovery method.

Table 5 - Bias, mean absolute error (MAE), mean square error (RMSE), adjusted coefficient of determination R^2_{adj} and Reynolds index in percent (*EI'*) for the three studied distributions and the two approaches of distribution modeling.

Method	Criteria	Weibull ML	Weibull MoM	Beta	Normal
Parameter estimation	Bias	0	0	0	0
	MAE	1.42	1.40	1.41	1.46
	RMSE	1.92	1.89	1.91	1.97
	R^2_{adj}	0.746	0.754	0.751	0.734
	EI'	36.96	37.07	39.90	37.55
Parameter recovery	Biais	0	0	0	0
	MAE	1.90	1.89	2.46	1.90
	RMSE	2.687	2.670	3.639	2.608
	R^2_{adj}	0.505	0.511	0.091	0.5333
	EI'	48.99	48.69	62.68	48.57

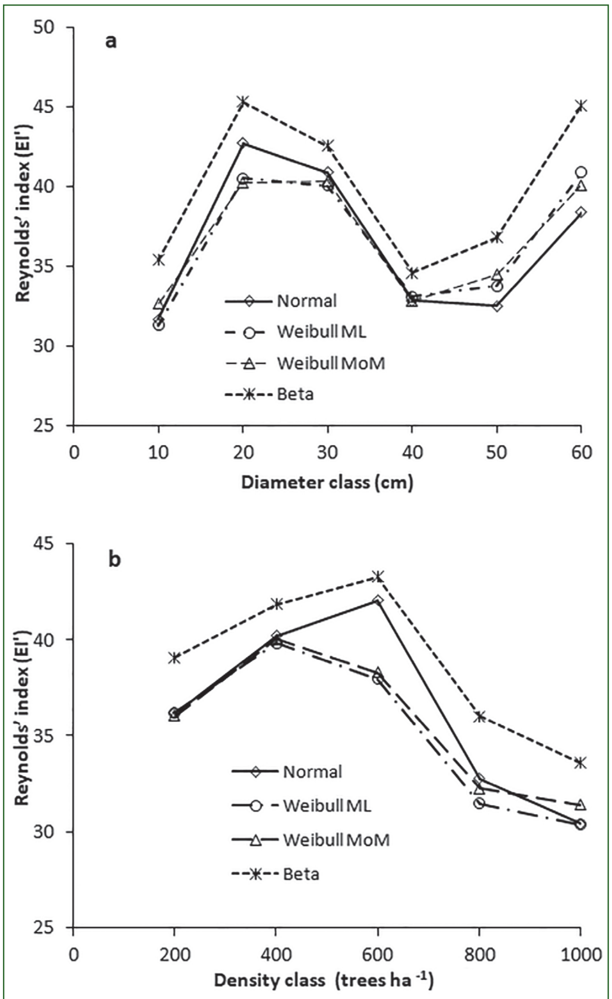
Parameter estimation method

Table 5 shows values of the different comparison criteria which are very close to each other despite a slight superiority of the Weibull distribution, which uses the method of moments as a parameter estimation method.

Figure S1 (in supplementary material) shows the distribution of residuals according to stem size; the normality test of residuals and the projection of residuals according to the normal scores for each of the studied distribution. From these graphs, it appears that residuals of all studied distributions are randomly distributed around the null value as a function of diameter and have distributions which do not deviate too much from a normal one, with ρ_{obs} values of 0.99 vs. a theoretical value of the order of unit ($\rho_{0.05}$ = 0.998).

Reynolds index (EI') evolution as a function of plots' mean diameter and mean density classes (Fig. 2a and Fig. 2b respectively) shows that the Beta distribution is the least accurate one comparatively to the others. This trend applies for all diameter and density classes. The accuracy of the two other distributions, i.e. the normal and the Weibull one with the two parameter estimation methods (ML and MoM), varied according to diameter and density classes: the two Weibull distributions were similar, while the normal distribution differed from the two Weibull ones by remarkably higher values of EI' for the diameter class of 20 cm and the density class of 600 stems/ha and by lower values for the two last diameter classes (50 and 60 cm).

Figure 2 - Mean values of Reynolds index per class of mean diameter (a) and mean density (b) (method of parameter estimation).



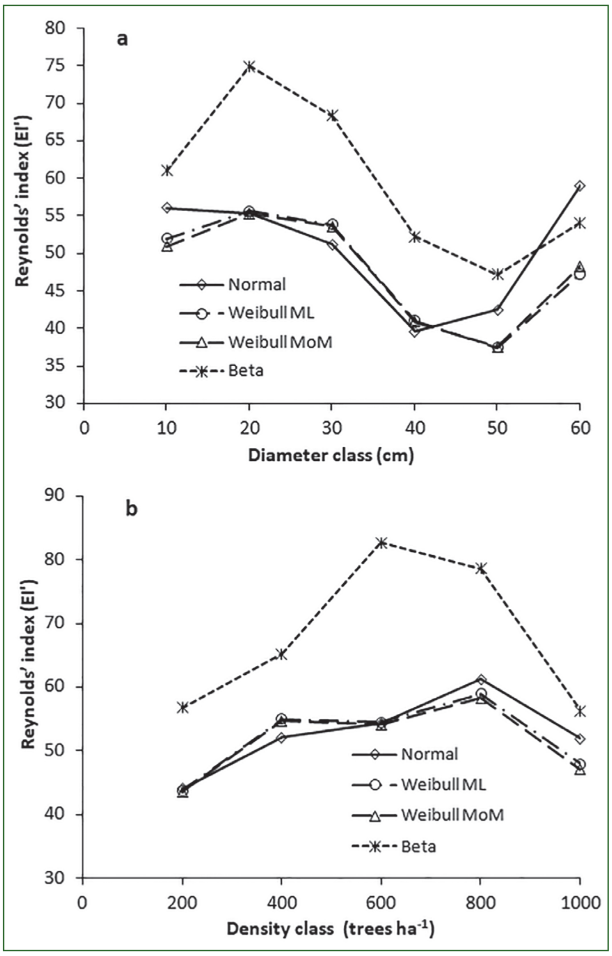
Parameter recovery method

This method showed (Tab. 5) that the Beta distribution differed by the highest values of the mean absolute errors (MAE), the root means square errors (RMSE), the Reynolds index (EI') and the lowest value of the adjusted coefficient of determination (R^2_{aj}). Such distribution is the least accurate one regarding prediction of the number of trees per diameter class. Values of the different comparison criteria for the three other distributions, namely the normal distribution and Weibull ones with the two estimation methods, are of the same order of magnitude even though they slightly favor, from a precision point of view, the normal distribution and the Weibull distribution estimated with the method of moments.

Figure S2 (in supplementary material) on the homoscedasticity and normality of residuals which resulted from the studied distributions, in the case of the parameter recovery method, shows that, in general, the distribution of residuals deviated much more from the normal distribution comparatively to the distribution obtained with the parameter estimation method. Indeed, values of ρ_{obs} related to Ryan-Joiner normality test ranged from 0.95, for the Beta distribution, to about 0.98 for the normal one. Those related to the Weibull distribution, with the two estimation methods, occupied an intermediate position with ρ_{obs} of 0.97. Regarding homogeneity of residuals distribution, around the zero value, along the horizontal axis, corresponding to diameter classes, only residuals from the Beta distribution revealed imbalanced for stem diameters lower than 20 cm, indicating an overestimation of the number of trees for small diameter classes.

Comparison of mean values of Reynolds Index (EI') by diameter classes (Fig. 3a) and density classes (Fig. 3b), in the case of parameter recovery method, confirmed results obtained in Table 5.

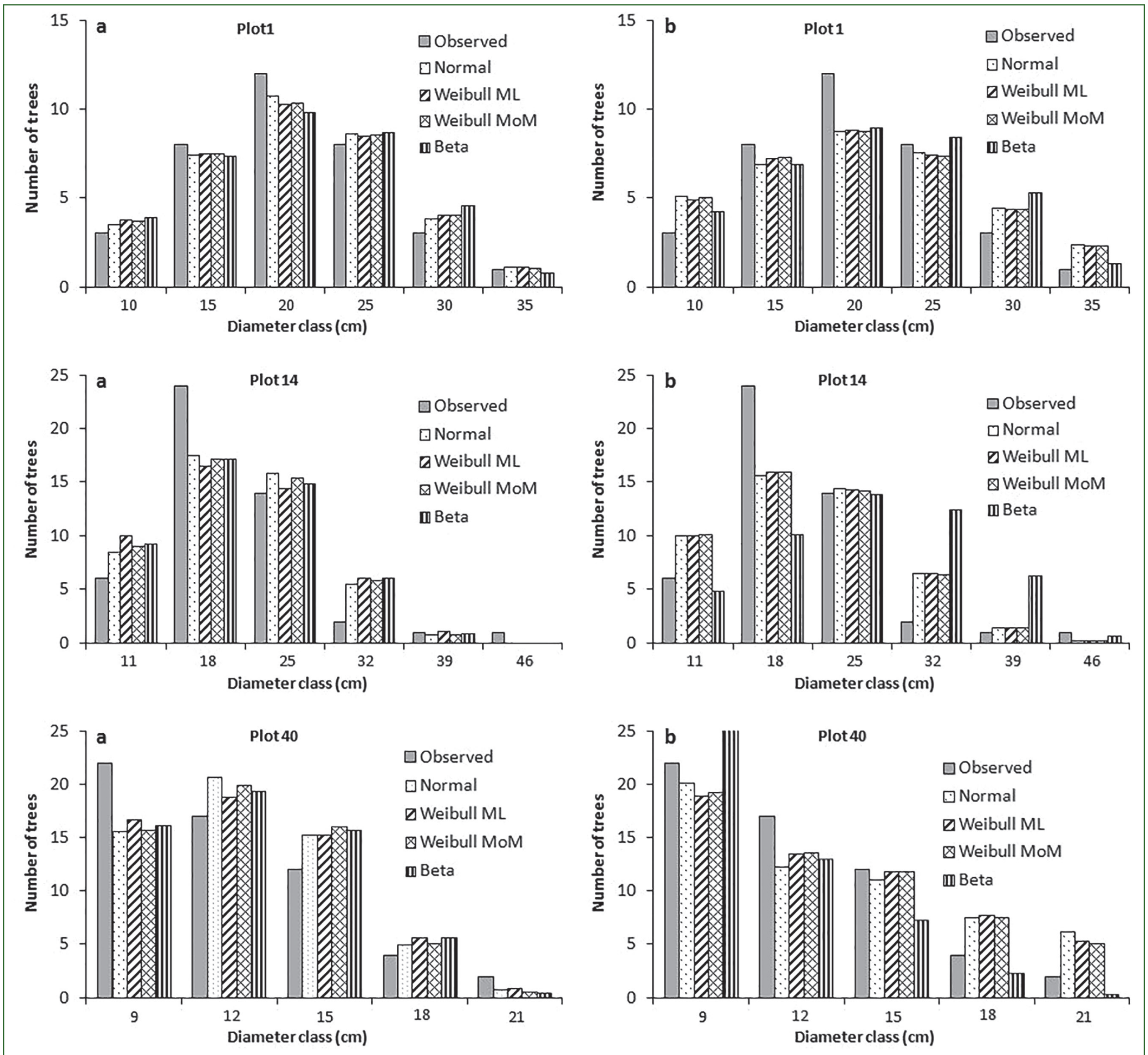
Figure 3 - Mean values of Reynolds index per class of mean diameter (a) and mean density (b) (method of parameter recovery).



Indeed, these two graphs show the accuracy difference between the Beta distribution and the other studied ones. Mean values of the Reynolds index obtained by the Beta distribution exceeded those obtained by the other distributions for all age classes and for different diameter classes except that of 60 cm for which the Beta distribution revealed more accurate than the normal one. The two distributions derived from the Weibull (ML and MoM) provided, in the case of parameter estimation method, values that are very close to each other according to plot distribution by diameter or density classes. In the case of parameter estimation method, the normal distribution is also different from Weibull ones for some diameter and density classes. Indeed, comparatively to the two Weibull distributions, despite a slight superiority for the diameter of 30 cm and the density class of 400 stems ha^{-1}, the normal distribution offers less accurate estimates, particularly for the first; the second and the last two diameter classes (i.e.10, 50 and 60 cm) and also for the two last density classes (800 and 1,000 trees ha^{-1}).

Three distributions shapes were revealed: symmetrical; dissymmetrical with left asymmetry and reverse *I* or *J*-shaped distributions. Figure 4 shows, for three representative plots of the three observed distribution types, the observed and the simulated numbers of trees by class of diameter according to the two methods of parameter calculation, i.e. the parameter estimation method (Fig. 4a) and the parameter recovery method (Fig. 4b). The graphs (Fig. 4) provide a further insight into the quality of tree size estimation by class of size using the studied distributions for each of the two parameter calculation method.

Figure 4 - Distribution of the number of stems per class of diameter, Observed and estimated numbers with the four studied functions basing on the method of parameter estimation (a) and the method of parameter recovery (b) for three characteristic plots (plot1: a symmetrical bell distribution; plot 14: a bell shaped left skewed distribution and plot 40: a reverse J distribution-shape).



Discussion

Results of the correlations between parameters of the theoretical distributions and those of the population, confirmed the flexibility of the Weibull distribution and its adaptability to various kinds of data (Rinne 2009), but are not in perfect agreement with those of Sghaier et al. (2016) on *Tetraclinis articulata* (Vahl) Mast. from Tunisia. The correlation coefficients calculated in the present study are lower than those found by Sghaier et al. (2016). On the other hand, the absence of correlation is not necessarily observed on the same stand variables as these authors. Considering only the Weibull distribution, results presently obtained by the two methods (ML and MoM) are comparable to those of Gorgoso et al. (2007), who used a nonlinear regression as an estimation method, if we except correlations with LP_{25} and Ld for the parameter *b* and N, H_{dom} and H_m for the parameter *c*. The same authors obtained on *Betula alba* L., in Spain, models explaining the Weibull parameters *b* and *c* with the same population variables (respectively d_g and P_{25}, in addition to N and H_{dom}) and with comparable coefficients of determination. Contrary to our results where the standard deviation is predicted by P_{75}, the best explanatory variable of this parameter, in the case of the normal distribution, is dg with a lower R^2 (0.526) according to Sghaier et al. (2016). Exceptionally high R^2_{adj} (up to 0.99) are obtained for models predicting the Weibull *b* and *c* parameters (Sghaier et al. 2016). With respect to Weibull distribution, multiple regression models established by Maltamo et al. (1995) on *Pinus sylvestris* and *Picea abies* (L.) Karst., in Finland, revealed different results regarding the explanatory parameters inputted and the quality of the adjustments with less explicit models ($0.18 < R^2 < 0,32$). On the other hand, more convincing results are obtained on the parameters of the Beta distribution. Comparisons made in the present study suggest the Weibull distribution as the most suitable. Although a large difference in error index is not observed, the method of moments seems to be the most recommended since it has shown satisfactory results regardless of the distribution shape. For Lejeune (1994), the normal distribution provided results as satisfactory as the Weibull distribution, despite a lower flexibility. For the Weibull distribution, the parameter estimation method by the non-centered moments proved to be the best, while there was not a large difference in adequacy between the Beta and the Weibull distributions estimated by the maximum likelihood method (Maltamo et al. 1995). Although most studies (Zhang et al. 2003 and references therein) indicate that the best adjustments are obtained by the maximum likelihood method, this one highly underestimates the small diameter frequencies what considerably reduces the quality of the obtained model. These findings corroborate those of Gorgoso et al. (2007). Regarding the method of moments, which also suffers from this disadvantage, it is cited among the most precise estimation methods (Lejeune 1994, Lei 2008, Liu et al. 2004, Liu et al. 2009). The weakness of this method consists in its difficulty to model the multimodal distributions. Nevertheless, other much less used procedures have shown satisfactory results (see for example the nonlinear regression method in Gorgoso et al. 2007). In agreement with these authors, results of the present study showed that establishment of a stem distribution model by parameter recovery method leads to inaccuracies comparatively to parameter estimation method. Lejeune (1994) attributed this loss of information to the unclear data from which the prediction equations are constructed. The author also considered that the step of parameter estimation constitutes the most important source of inaccuracy in the distribution modeling whatever the theoretical distribution adopted. This observation is due to the close relationship between the precision of this estimation and the number of individuals in different samples.

Results show strong correlations between parameters of the theoretical distributions and some population variables such as arithmetic or quadratic mean diameter and dominant height. For the same species, Palahi et al. (2006) showed that the quatratic mean diameter and the number of trees per hectare are the best predictors for the two parameters of the Weibull distribution (*b* and *c*, respectively). The diameter distribution models can be used independently by measuring a number of stand variables, or together with the yield table, which provides information that allows to predict the parameters of the distribution employed (work not yet published). Table 6 presents the predicted Weibull PDF (probability density function) parameters for different ages of the best site quality (H_d = 19.5 m at 70 years of age). Figure 5 shows the curves associated with these distributions.

Figure 5 - Curves of diametric distribution at 40, 70, 60, 100 and 120 years by diameter (DBH, cm) for stands.

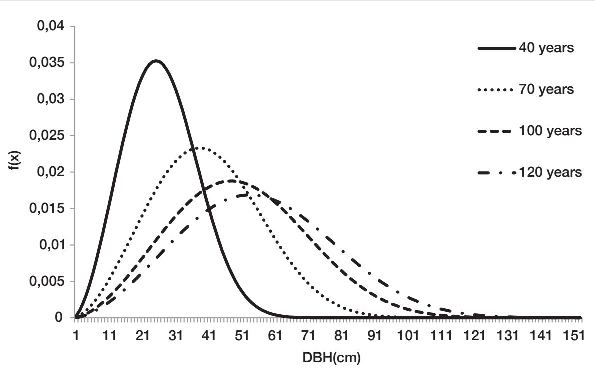


Table 6 - Values of Weibull PDF parameters at 40, 70, 100 and 120 years of age for stands.

	Stand variables					Estimated variables		Parameters Weibull PDF	
Age	H_d	H_m	N	C_g	d_g	$\bar{d}$	P_{25}	b	c
40	14.72	12.97	427.25	93.07	29.64	26.86	27.92	29.97	2.65
70	19.50	17.18	260.25	140.78	44.83	41.31	40.77	45.57	2.66
100	22.59	19.90	192.90	177.46	56.52	51.72	50.02	56.82	2.68
120	24.06	21.20	166.55	197.37	62.86	57.79	55.42	63.38	2.68

Conclusion

The present study aimed at building a model of diameter structure for Aleppo pine stands in the Aures (Algeria). Results indicate that the Weibull distribution is more suitable to this type of data with the method of moments as the best parameter estimation method. This study show that the parameter estimation method is more accurate compared to parameter recovery method. The parameter prediction models developed enable one to predict the diameter distribution for a given stand of *P. halepensis* using rather limited information (age, H_d, H_m, d_g, $\bar{d}$...). This model can be used to predict stand development under different management parameters. This information is crucial for finding optimal forest management schedules for different management objectives and stand conditions (Palahi et al. 2006).

Establishment of a structural model for the Aures Aleppo pine forests is an important and complementary tool to the previous investigations on silviculture and productivity of the species in the same forests (Bentouati 2005). Such tool may also help foresters to have an accurate idea on the woody material in a climato-ecological context requiring caution in silvicultural practices in order to maintain the balance and the sustainability of forest stands. These results are also useful in establishing a yield table by diameter classes which may be used as a management tool of Aleppo pine forests in the Aures which are important both ecologically and economically.

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