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Research paper

Comparison of TLS against traditional surveying method for stem taper modelling. A case study in European beech (*Fagus sylvatica* L.) forests of mount Amiata

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ABSTRACT Traditionally, taper equations are developed from measurements collected through a destructive sampling of trees. Terrestrial laser scanning (TLS) enables high levels of accuracy of individual tree parameters measurement avoiding tree felling. With this study, we wanted to assess the performance of two approaches to calibrate a taper function: using stem diameters extracted from TLS point clouds and measured at different tree heights with the traditional and usual forest instruments. We compared the performance of four taper equations built with data collected by TLS and traditional survey in a European beech (*Fagus sylvatica* L.) forests of mount Amiata (Tuscany Region, Italy). We computed the volume of stem sections 1.00 m long by integrating the most performing TLS-based taper equation and by the Huber, Smalian and cone formulas applied on the diameter and height values measured with the traditional field surveys. We conducted the analysis of error distribution in volume estimates computed integrating the most performing TLS-based taper function along the stem. We tested if the differences in the volume estimate of the two methods were significant. Schumacher and Hall (1933) equation was the most performing taper function both in case of using TLS and traditional surveyed data, being the TLS-based function more performant (rRMSE = 6.90% vs 9.17%). Its performance did not increase when diameter values were extracted from TLS point clouds with a higher frequency (i.e. 25.0 cm vs 1.00 m). By integrating the TLS-based Schumacher and Hall (1933) function, the sections with the highest error resulted from 5.00 to 7.00 m of stem height (i.e. RMSE from 14.72 to 19.14 dm^3 and rRMSE from 13.00 to 17.76%). This study case represents the first attempts to develop a taper equation for European beech of mount Amiata using values of stem diameter and height extracted from the TLS point cloud. The results demonstrated that TLS produces the same stem volume estimates as traditional method avoiding falling trees.

KEYWORDS: terrestrial laser scanner, log stem volume, wood assortments, taper function.

Introduction

Taper functions, taper equations or taper curves describe the profile of tree stems mathematically and enable the estimation of the diameter of a tree stem at any tree height of interest and vice-versa the inverse estimation, i.e. at which height the stem has a given diameter. For sake of clarity, the profile represents the characteristic shape of a vertical cross-section of the stem, whereas taper refers to the rate of decrease in stem diameter with increasing height from ground level to the treetop (Burkhart and Tomé 2012). Taper functions provide estimates of inside or outside bark diameter at any point along the stem, total stem volume, merchantable volume, merchantable height to any top diameter and from any stump height, and individual log volumes of any length at any height from the ground (Kozak 2004). From a commercial point of view, they offer the possibility to determine the type of merchantable products obtainable from a stem (Corona and Ferrara 1987) and to estimate the total volume for each product or wood assortment. From a silvicultural point of view, taper equations allow also to assess the effects on stem taper of environmental conditions and genetics (Gomat et al. 2011) and social position of a tree within the stand (Das and Awadhiya 2006). In plantation forestry of commercially important tree species, taper equations are used to investigate the effects of stand density (Duan et al. 2016, Ramalho et al. 2019), thinning (Mäkinen and Isomäki 2004, Weiskittel et al. 2009), pruning (Fernández et al. 2017). Providing detailed depiction of average relative stem profiles, taper curves can also be used to analyze the long-term effect of fertilization on stem form, taper and volume (Snowdon et al. 1981, Bi and Turner 1994). Besides, the knowledge of form and taper can support studies on the statics of stems and their resistance to mechanical strains (Papesch et al. 1997).

Since the first half of the twentieth century, several types of equations have been developed. Burkhart and Tomé (2012) divide taper equations into three major categories according to the structure of equations: simple equations, segmented equations, variable-exponent equations. An example of simple equations is the parabolic equation developed by Kozak et al. (1969) for the prediction of the tree profile inside bark and the trigonometric based function developed by Thomas and Parresol (1991). Other examples of simple equations include that of

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Ormerod (1971), Laasasenaho (1982), Reed and Byrne (1985), Pain and Boyer (1996), and Sharma and Oderwald (2001). Segmented equations are based on the principle that tree stem can be approximated to various geometric solids: for example, the lower bole portion is generally assumed to be as or approximated by a neiloid frustum, the middle portion to or by a paraboloid frustum and the upper portion to or by a cone. Consequently, for any part of the stem, an equation is developed and unique model results from the join of the single equation, which can be fitted using segmented regression techniques. Researches on segmented functions have been carried on by Max and Burkhart (1976), Brink and Gadow (1986), Clark et al. (1991), Gadow and Hui (1999), Sharma and Burkhart (2003), Brooks et al. (2008), and Cao and Wang (2015). Variable-exponent equations are simple continuous functions that describe the shape of tree boles with a varying exponent from the ground to treetop to account for neiloid, paraboloid and conic forms. Newberry and Burkhart (1986), Newnham (1988) and Kozak (1988) were the first developers of this functions, then a variety of approaches have been developed, for example, by Newnham (1992), Flewelling and Raynes (1993), Kozak (1997), Kozak (2004) and Sharma and Zhang (2004).

Taper equations are species-specific because the shapes of the stem of different species may differ appreciably. It means that applying a stem taper function developed for one species to another species could lead to substantial bias in diameter and consequently in volume estimates. Taper equations are developed for particular tree species in the forest of major commercial interest and try to guarantee the representativeness of local conditions. Therefore, for each species, a separate set of parameters is needed for a fixed taper equation that identifies the unique bole shape. For these reasons, forest scientists have developed many taper functions for many tree species and often for particular regions where a species grows. In Canada, Solomon et al. (1989) and Yang et al. (2009) developed taper models for spruce (*Picea* spp.), and Sharma and Zhang (2004) and Lejeune et al. (2009) developed variable-exponent taper equations for jack pine (*Pinus banksiana* Lamb.) and black spruce (*Picea mariana* (Mill.) Britton, Sterns and Poggenb.), respectively. In the USA, Sharma and Oderwald (2001) developed a taper function for trees in natural forests of loblolly pine (*Pinus taeda* L.) in the southern USA, Garber and Maguire (2003) built taper functions for ponderosa pine (*Pinus ponderosa* P. Lawson and C. Lawson), lodgepole pine (*Pinus contorta* Dougl. Ex Loud.) and grand fir (*Abies grandis* (Douglas ex D. Don) Lindl.) applicable to the full range in tree size and density of stands in central Oregon. In Europe, Laasasenaho (1982) built taper curves for pine, spruce and birch species growing in Finland, equations for poplar (*Populus* spp.) growing on farmland in Sweden were developed by Hjelm (2013). Calama and Montero (2006) built taper functions for stone pine (*Pinus pinea* L.) in Spain and Rodriguez et al. (2015) for the eight major tree species in the Spanish Plateau. In Italy, Scrinzi and Tabacchi (1979) mathematically interpreted the rate of decrease in stem diameter with increasing height of Norway spruce (*Picea abies* (L.) H. Karst.) growing in Centro-Cadore (North East of Italy), Corona and Ferrara (1987) developed taper function for Monterey pine (*Pinus radiata* D. Don) growing along the peninsula, and Scotti et al. (2014) developed taper curve for maritime pine (*Pinus pinaster* Aiton.) growing in Sardinia.

Traditionally, the development of taper equations requires the trees felling and therefore a destructive sampling. As Bortolotto Buck et al. (2019) highlight, non-destructive approaches of tree scaling, which relate different measurable properties of trees to each other, constitute a relevant advance for forest mensuration. Within these approaches, TLS represents a suitable technique for detailed data collection. The dense three-dimensional point cloud allows characterizing an individual tree with a millimetric level of detail enabling a highly detailed inventory of tree parameters. A modern TLS device can measure 10^4–10^6 points per second with an accuracy ranging from 0.1 cm to 1.0 cm (Oguchi et al. 2011). In forest inventory, TLS is used to estimate tree attributes, e.g. diameter at breast height (i.e. 1.30 m at the tree height, DBH) (Torresan et al. 2018, Pitkänen et al. 2019), tree height (Liu et al. 2018), crown base height and average crown spread (Mohammed et al. 2018). Branch and stem volume (Dassot et al. 2012), biomass (Calders et al. 2015, Srinivasan et al. 2015), stem volume (Bortolotto Buck et al. 2019) and volume of the parts of a trunk with certain radii and the branches of a certain size (Liang et al. 2016) are computed using TLS data. Several studies have developed taper function using TLS data avoiding labour-intensive field operations and destructive stem analysis. Among those, Liang et al. (2014) used TLS to produce stem curve for Scots pine (*Pinus sylvestris* L.) and Norway spruce, obtaining a Root Mean Squared Error (RMSE) equal to 1.13 cm versus 1.03 cm in the case of manual stem curve measurements. Sun et al. (2016) extracted stem diameter at different tree heights from TLS point clouds of a poplar cultivar (*Populus x canadensis* Moench) and established the relationship between point cloud data to developed localized taper functions. In this case, the RMSE of the six fitted taper equations developed using TLS data ranged from 7.43 cm to 11.43 cm while using traditional destructive measurements errors ranged from 7.80 cm to 11.93 cm. In both cases, tree TLS-based measurements were acquired at the plot level, with a radius of 10 m and 15 m and using a multi-scan approach, from 6 and 4 locations in the case of Liang et al. (2014) and Sun et al. (2016) respectively.

We are not aware of researches aimed at developing taper equations for European beech (*Fagus sylvatica* L.) using TLS data. European beech is the major broad-leaved tree species in central and western Europe (EUFORGEN 2008) but it also extends further south into the Mediterranean basin where it is confined to mountainous regions (García-Plazaola and Becerril 2000). The presence in southern Europe is the result of the diffusion following the last ice age during which the European beech found refuge in the Balkan regions, in Italy and Spain. Postglacial recolonization came very slowly, following the oaks (*Quercus* spp.) and the silver fir (*Abies alba* Mill.), assuming dominance only 2,000-3,000 years after the first appearance as a sporadic species (Mayer 1977). In Italy, according to the last national forest inventory (INFC 2005), European beech high forests cover 12% of the total surface of high forests. Beech is an important and widely used hardwood in Europe: its hardness, wear-resistance, strength, and excellent bending capabilities, coupled with its low price, make this hardwood a mainstay for many European woodworkers. Depending on soil conditions, European beech can grow to very large sizes, and wide, long lumber is commonly available for use (Meier 2015).

Therefore, based on previous studies we hypothesize that, even in the case of European beech, the performance of stem taper models developed using TLS data are at least equivalent to the performance of models developed with data collected with a traditional destructive method. We also hypothesize that, to build the stem taper model, the option to inexpensively perform a more intense sampling of the diameters along the profile using TLS than that realized by field surveys could increase the performance of the taper function. Finally, we hypothesize that error of the volume estimates obtained by the integration of the TLS-based taper function is related to the height of the tree. To test these hypotheses, the study was conducted with the overall goal of comparing two different approaches for wood volume assortment estimation: the destructive traditional approach based on tree felling and along with stem diameter measurements versus the approach based on close-range sensing. Specific objectives were: *(i)* identify the most performant TLS-based taper function among a set of selected equations and compare its performance with the performance of the field-based taper equation; *(ii)* analyse whether the performance of the most performing TLS-based taper function increases using diameter values extracted from the TLS point cloud with a frequency higher than that traditionally used in field survey; *(iii)* assess the distribution of the differences in volume estimates computed by the integration of the most performing TLS-based taper equation in comparison to the volume estimates computed using traditional formula.

Materials and methods

Mount Amiata study area and European beech forest stand

The study area is the mount Amiata located in Southern Tuscany (Italy). The recent volcanic origin of mount Amiata, which assures high site fertility (Selvi 1996), the annual rainfall of 1,547 mm, concentrated in autumn and winter, and the mean annual temperature of 10 °C make the right combination of conditions for the growth of European beech forests with high economic, environmental and social value. For our research, we considered a European beech forest stand located in the northern slopes of the mount Amiata at around 1,200 m a.s.l. administratively located in two municipalities, Seggiano and Abbadia San Salvatore in Grosseto and Siena provinces respectively. The silvicultural approach applied in the stand, composed of trees aged from 60 to 80 years, is characterized by progressive thinning from below, finally aiming at the regeneration cut carried out by shelterwood system.

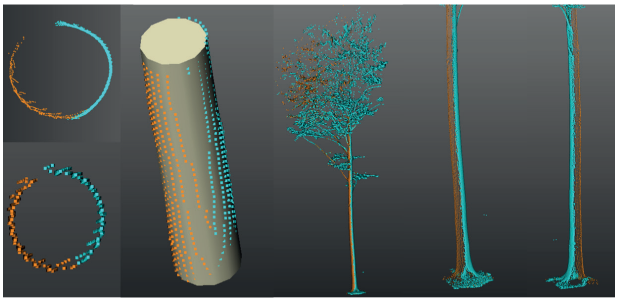
Sampling and terrestrial laser scanning survey

In spring 2017, in the leaves-on period, a sample of trees, needed to build up the taper functions, was subjectively selected to represent all main tree diameter and height sizes. Attention was paid to ensure that none of the chosen trees had damaged tips, stem defects, forks, or injuries that led to stem rot. Moreover, only trees with free crown were chosen. In total, 34 trees were selected. Each tree was marked with a unique identifying code to be easily identified from afar. If needed, pre-scan preparations were made to clear the undergrowth and remove lower branches and shoots. The scanning activities were conducted in the clear sky and windless conditions using a Focus3D X 130 (FARO Technologies Inc., Lake Mary, FL, USA) whose features are reported in Torresan et al. (2018). Each tree was scanned from two opposite positions having located six 140 mm-diameter spheres needed for the co-registration of the two point clouds. This target type has been proven to be the most effective laser scanning target for co-registration because a spherical shape allows for the highest possible scanning efficiency from various directions and always provides a homogeneous reference surface (Brazeal 2013). The targets were placed on poles at different heights and distributed around the tree. The Focus3D X 130 laser scanner was positioned at a distance between 8 m and 10 m after checking the visibility of the entire tree from the point where the instrument was positioned and the visibility of the 6 spheres. The field of view was set at an angle of 120° in the horizontal direction and at 310° in the vertical direction to limit the time needed for a scan.

We decided to used two scan stations around the single tree because according to Saarinen et al. (2017) two TLS point clouds combined for deriving stem information from single trees allow more accurate results in volume estimation than the single scan.

Specifically, the results of their study indicated that, in the case of two scans per tree, the accuracy of stem volume estimates improved decreasing the RMSE from 12.4% to 6.8%. This type of configuration allowed to cover the entire tree height (Fig. 1).

Figure 1 - Example of a single tree TLS point cloud resulting from two separated scans (orange and light blue points). In (c), the points along a portion of a stem. In (a) and (b) the points around the circumference of the lower and upper part of the stem portion. In (d), the whole TLS point cloud of the scanned tree. In (e) and (f), the points clouds obtained from two opposite scans.



Traditional measurements of felled trees

At the end of the TLS survey, every scanned tree was felled in the manner that minimized stem breakage. After felling, measuring tape was stretched along the bole and tree length from the base of the stump to the top of the tree (H, to the nearest 0.01 m) and DBH (to the nearest 0.1 cm) were recorded. Successively, to avoid the irregularity of the base of the stem, the first diameter over bark was measured at 0.50 m (to the nearest 0.1 cm), then every 1.00 m till the stem reached a value of the diameter at least of 10.0 cm in the upper part of the trunk. All diameters at height h were measured with a calliper along two perpendicular axes in north-south directions and successively the two measures were averaged.

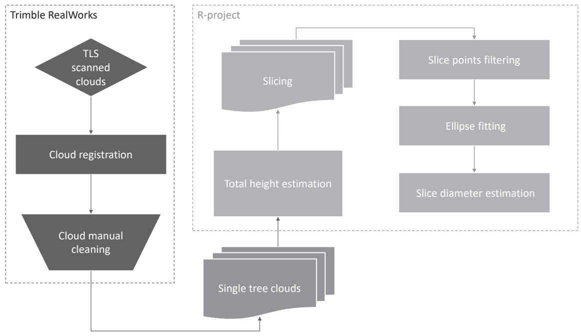
Terrestrial laser scanning data processing

The complete flow chart of TLS data processing, aimed at estimating observed stem diameter at all selected heights, is reported in Figure 2.

For each tree, the two scans were registered into a local coordinate system by the reference sphere method using Trimble RealWorks Software (Trimble Inc., Sunnyvale, CA, USA). Using the same software, manual TLS data cleaning was performed to obtain the cloud with all points belonging to a single tree. For each tree point cloud, the total tree height was calculated as the total vertical extent of the cloud itself.

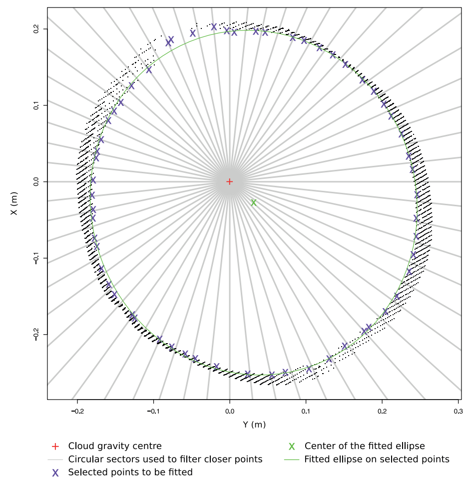
The rest of the processing has been performed using the R software (R Core Team 2020) and specifically the packages lidR (Roussel and Auty 2019) and

Figure 2 - TLS data processing flow chart.



conicfit (Gama and Chernov 2015). Considering 25.0 cm step along the tree height, the point cloud was divided into slices of 15.0 cm of thickness. It is known that the vertical thickness of the TLS point cloud slice influences the accuracy of the diameter measurements (Xie et al. 2020). At the same time, the thicker the slice thickness, the more point cloud number from which to extract the diameter will be and the more time for data processing will need. Besides, as the step influence the number of obtainable slices, we balanced with these two aspects considering that a thickness of 15.0 cm still allows reaching a good accuracy in the diameter estimation (Xie et al. 2020). Starting from the stump, each slice was cleaned by an automatic algorithm that identifies the centre of gravity of a slice and, for circular sectors of 6°, selects the point closest to the centre. Then, an ellipse was fitted on filtered cloud (Fig. 3).

Figure 3 - Example of the result from the TLS point slice cleaning process which identifies the centre of gravity, selects the closest points to this centre and fits an ellipse in the filtered cloud.

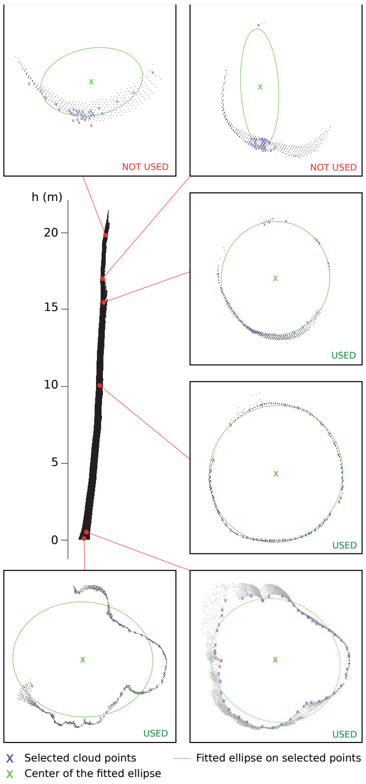


For every step, the two ellipse radii (r_1 and r_2) were obtained and the diameter of the stem was computed as in (1):

$$d = 2\,\frac{(r_1+r_2)}{2} = r_1+r_2 \qquad (1)$$

Slices, where sectors have less than 6 points, were skipped because the points were not enough to fit an ellipse. The steps above described were repeated in the successive slice. The process stops when no points being detected. All fitted ellipses have been plotted, together with the corresponding cloud points, to detect and exclude anomalous cases, i.e. real cross-section insufficiently represented by the available cloud points (Fig. 4).

Figure 4 - Examples of the fitted ellipses on selected points used and discarded from the analysis.



Stem diameters observed at the heights along the bole corresponding to the manual measurements (i.e. every meter starting from 0.5 m) represent the main sample. Additional diameters, i.e. every 0.25 m, have been used to test a specific hypothesis.

Stem taper models considered in this evaluation

From the literature, four equations were selected to model the stem taper of European beech of mount Amiata. The four equations were chosen because they are the most used in several species and forest types around the world (Hjelm 2013, Beltran et al. 2017, Doyog et al. 2017, Poudel et al. 2018, Sakici and Ozdemir 2018) and have been already utilized with terrestrial laser scanner data (Sun et al. 2016).

The equations are used to estimate diameters (d) of the stem at any height (h) and were designed to meet the condition that d = 0 when h = H, where H is the treetop height. a_0 to a_n and b_0 to b_n are function parameters and DBH is the diameter at breast height.

The equations are listed below:

$$d=\sqrt{a_0\,DBH^{a_1}\,\frac{(H-h)^{a_2}}{H^{a_3}}}$$ (2) Schumacher and Hall (1933)

$$d=\sqrt{DBH^2\,\frac{(H-h)^{a_0}}{H-1.3}}$$ (3) Ormerod (1971)

$$d=\sqrt{a_0\,DBH\,\frac{(H-h)^{a_1}}{H-1.3}}$$ (4) Yan (1992)

$$d = a_0\,DBH^{a_1}H^{a_2}\left(\frac{1-\left(\frac{h}{H}\right)^{\frac{1}{3}}}{1-\left(\frac{1.3}{H}\right)^{\frac{1}{3}}}\right)^{\left(b_1\left(\frac{h}{H}\right)^4+b_2\left(\frac{1}{e^{\frac{DBH}{H}}}\right)+b_3\left(\frac{1-\left(\frac{h}{H}\right)^{\frac{1}{3}}}{1-\left(\frac{1.3}{H}\right)^{\frac{1}{3}}}\right)^{0.1}+b_4\left(\frac{1}{DBH}\right)+b_5H^{\left(1-\left(\frac{h}{H}\right)^{\frac{1}{3}}\right)}+b_6\left(\frac{1-\left(\frac{h}{H}\right)^{\frac{1}{3}}}{1-\left(\frac{1.3}{H}\right)^{\frac{1}{3}}}\right)\right)}$$ (5) Kozak (2004)

Model fitting, validation, and comparison of the predictive performances

Data required to build taper functions include tree characteristic (i.e. DBH, height) and profile data (i.e. series of diameter measurements along the bole at given heights). Profile data from the same tree are naturally much more correlated than measurements from different trees: this type of data structure leads to the autocorrelation problem, a situation where a certain level of dependence is observed among different data (Senyurt et al. 2017). Autocorrelation may lead to systematic errors in confidence interval estimations during the calculation of stem taper equation parameters, although these remain unbiased and consistent (Diggle et al. 2002), which in turn can negatively affect the reliability of the model results (Searle et al. 1992). As a result, statistical tests using t or F distributions and resulting inferences based on the independence of residuals are not reliable, because estimates of the residual covariance matrix are biased (Kublin et al. 2013, Tang et al. 2016). To face this issue, we assumed that the correlation of within-tree measurements decreases with the distance, that is, the height of the tree at which the diameter measurement is taken. Therefore, the structure of an autocorrelated error was imposed using a first-order autoregressive correlation structure (Pinheiro and Bates 2000). Along with autocorrelation, the collected sample data resulted also affected by heteroskedasticity, which would lead to non-constant error variance, i.e. unreliable predictor intervals. Among others, a solution to mitigate heteroskedasticity is to weight each observation during the fitting process. According to Pinheiro and Bates (2000), we applied a variance function that assigned a fixed weight to the DBH covariate (i.e. the square root of DBH). To solve the aforementioned issues, we used the gnls() function of the *nlme* package (Pinheiro et al. 2020) of the R software. This function fits nonlinear models, using generalized least squares, allows the errors to be correlated, and manages the data with unequal variances. Finally, to compare the goodness of fit of candidate models, and hence to choose the best stem taper equation, a likelihood ratio test was conducted to detect which model exhibited the most significant reduction in overall deviance.

When the most performing model was detected, as a form of model validation, to mitigate the overfitting due to the use of all available data in model building, the Leave-One-Out Cross-validation (LOOCV) was used. This procedure iteratively uses a single higher-level observation, i.e. a single tree, for the validation, and the remaining observations as the training data. The step is repeated so that each observation in the sample is used once as validation data. Mean Bias Error (MBE) and RMSE were calculated in the validation data set as in formula (6) and (7) together with the coefficient of determination (R^2) to compare the predictive performance of the models:

$$MBE = \frac{1}{n}\sum_{i=1}^{n} (\hat{d}_i - d_i) \quad (6)$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (\hat{d}_i - d_i)^2} \quad (7)$$

where $\hat{d}_i$ is the predicted value of the diameter for i-th section and d_i is the observed value for the same i-th section. The relative errors of the MBE and RMSE (rMBE and rRMSE respectively) were calculated as the ratio between the corresponding error value and the mean observed value.

The most performing TLS-based model underwent further analysis to assess whether its performance increases using diameter values extracted from TLS point cloud with a higher frequency than that usually used in the field, i.e. every 25.0 cm instead of 1.00 m. The same statistical analyses, i.e. model fitting and validation as described above, were carried out using the values of stem diameter extracted from the TLS point clouds with the higher frequency.

Stem and log volume estimation

In the traditional approach, tree stem and log volume are estimated by sectional methods: the volume is calculated based on cross-sectional areas, measured either at log midpoints or at log heads. In our study, the volume of the first log starting from the ground was calculated using the Huber's formula to avoid the errors caused by the root reinforcements. The volume of the subsequent logs, until a stem height with at least 10.0 cm of diameter, depending on the value reached at the whole meter on the trunk., was calculated using the Smalian's formula. The volume of the top section was determined using the cone formula (Tab. 1).

Table 1 - Formulas used to estimate the volume (V) of the sampled felled trees by traditional sectional methods.

Formula	Equation	Variable meaning	Portion of the stem
Huber	$V=S_{(1/2)} * l$	$S_{1/2}$ = cross-sectional area at the midpoint of the log length l = length of the log equal 1 m	First log, from the ground
Smalian	$V= (S_l+S_u)/2 * l$	S_l = cross-section area at lower of the log S_u = cross-section area at upper of the lon l = length of the log equal 1 m	From the second log to a stem height with 10-15 cm of diameter
Cone	$V=S_b/3 * (H H\_b)$	S_b = cross-section area at the base of the top H = total height H_b = stem height at the base of the top	End log toward the treetop

When a taper function is available, the volume of the whole stem or of any given portion can be computed by integration, summing up the volumes of thin cross-sections. For that, in the non-destructive approach, the volume of the logs has been computed integrating the most performing TLS-based model.

For comparison of the two approaches, the TLS-based volume estimates were compared with the traditional field-based volume estimates, considered as reference values, for logs 1 m long from 1.00 m up to 9.00 m along the stem. In the Italian forest sector, the wood assortments consisting of trunks with a useful length of 4.00 m and an average minimum diameter of 18.0 – 20.0 cm are classified as logs. This length guarantees trunk sufficiently cylindrical with woody fibre structure that makes them suitable for sawing. Following this commercial classification for wood merchantable products, our analysis was carried out in the portion of stem from 1.00 m to 9.00 m which corresponds to two logs.

TLS-based volume estimates were regressed versus reference section volumes, i.e. computed with values measured by traditional surveying method, and statistical regression metrics (i.e. R^2, adjusted-R^2, RMSE, standard error) were calculated.

Statistics to evaluate the error distribution along the stem were calculated as in (6) and (7), using as reference value the volume of the stem section calculated based on traditional formulas (i.e. Huber, Smalian and cone) and as predicted value the volume of the stem section estimated with the most performing TLS-based taper curve equation. Also, we tested if the volume of the log sections computed with the traditional approach and the volume of the same sections obtained from the integration of the most performing TLS-based model was not different. To do that, a test for the equality of distribution (i.e non-parametric Kolmogorov-Smirnov test) has been applied for each log section.

Results

Summary of dendrometric parameters collected using traditional surveying methods

According to the values of dendrometric parameters collected with the traditional surveying methods for stem taper modelling, the minimum, average and maximum DBH of 34 trees measured in the field were 21.0 cm, 41.2 cm and 67.5 cm respectively, while the minimum, average and maximum height was 20.90 m, 29.80 m and 37.70 m respectively.

Comparison of the predictive performance of taper equations

In Table 2 the Akaike (Akaike 1974) and Bayesian (Schwarz 1978) information criteria, AIC and BIC respectively, along with the likelihood-ratio test are reported, since lower AIC or BIC implies either fewer explanatory variables, better fit, or both. All this information helped us to rank the stem taper equations fitted using data collected both by traditional surveying and by TLS method.

A p-value lower than 0.05 indicates that there is statistical evidence to reject the null hypothesis that compared models fit likewise. As can be noted, the Schumacher and Hall (1933) fits the data better than the other models, both in the case of field-based and TLS-based data.

The parameter estimates, standard error values, confidence interval and their significance levels for Schumacher and Hall (1933) field-based and TLS-based equations with the success criteria are all shown in Table 3. All parameters for these equations were determined to be statistically significant with p-value < 0.05. MBE, and consequently rMBE, resulted slightly lower in the case of the field-based model, while RMSE and rRMSE were lower in case of TLS-based model (RMSE, 3.04 cm versus 2.29 cm and rRMSE 9.17% versus 6.90%).

In Figure 5, the values of stem diameter predicted by the Schumacher and Hall (1933) TLS-based function in relation to the tree height and their corresponding 95% confidence interval are plotted. In the same figure, the values of diameter measured in the field are superimposed to the predicted values: using the TLS approach as the generator of the reference values, roughly two third (65.87%) of field-based measures lay within the confidence intervals (see black dots).

Table 2 - Model performance statistics of the 4 equations analysed in the study (#eq. is the number of the equation as reported in the manuscript) resulted from the likelihood ratio test applied to the field-based and TLS-based models. The table shows the degrees of freedom (df), the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), the log-likelihood (logLik) and the likelihood ratio statistic (L.Ratio) with the associated p-value.

#eq.	Author	df	AIC	BIC	logLik	Test	L.Ratio	p-value
				Field-based model				
2	Schumacher and Hall (1933)	6	2753.36	2779.43	-1370.68			
3	Ormerod (1971)	3	2794.19	2807.23	-1394.09	2 vs 3	46.83	<.0001
5	Kozak (2004)	11	3019.18	3066.98	-1498.59	3 vs 5	208.99	<.0001
4	Yan (1992)	4	3384.71	3402.10	-1688.36	5 vs 4	379.54	<.0001
				TLS-based model				
2	Schumacher and Hall (1933)	6	2480.11	2506.19	-1234.06			
3	Ormerod (1971)	3	2549.25	2562.29	-1271.62	2 vs 3	75.14	<.0001
5	Kozak (2004)	11	3292.63	3340.43	-1635.31	3 vs 5	727.38	<.0001
4	Yan (1992)	4	3327.18	3344.57	-1659.59	5 vs 4	48.56	<.0001

Table 3 - The parameter estimates with the goodness-of-fit statistics for the field-based and TLS-based Schumacher and Hall (1933) models.

	Schumacher and Hall (1933)							
	Field-based model				TLS-based model			
Predictors	Estimates	Standard error	Confidence interval	p-value	Estimates	Standard error	Confidence interval	p-value
a_0	1.8618	0.3163	1.2419-2.4816	<0.001	2.1700	0.2666	1.6475-2.6925	<0.001
a_1	1.8367	0.0282	1.7814-1.8919	<0.001	1.8958	0.0217	1.8533-1.9384	<0.001
a_2	1.4179	0.0331	1.3526-1.4832	<0.001	1.3035	0.0249	1.2548-1.3522	<0.001
a_3	1.3944	0.0683	1.2606-1.5281	<0.001	1.3832	0.0497	1.2857-1.4806	<0.001
Observations		**34**						
MBE		-0.0029 cm			-0.0086 cm			
rMBE		0.0088%			-0.0260%			
RMSE		3.0377 cm			2.2860 cm			
rRMSE		9.1673%			6.9034%			
R^2		0.9574			0.9752			

Figure 5 - Predicted stem diameter values by the Schumacher and Hall (1933) TLS-based function in relation to tree height and their corresponding 95% confidence interval with superimposed the values of stem diameter measured in the field.

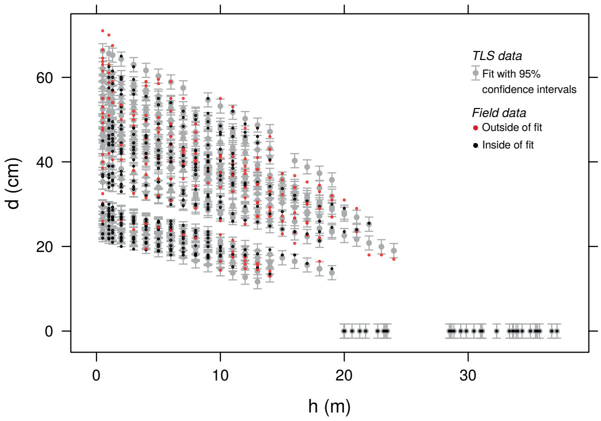
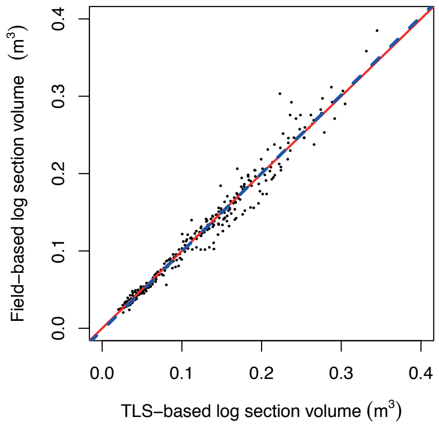


Figure 6 - Volume of sections 1 m long from 0 m to 9 m of trunk height estimated by integrating Schumacher and Hall (1933) TLS-based function against the volume computed with traditional formulas (fitted regression line is the blue dashed line and the 1:1 relationship line is the red solid line).



Exploiting the full potentiality of TLS by using the diameter extracted every 0.25 m along the stem does not raise the performance of the Schumacher and Hall (1933) taper equation (Tab. 4), increasing both the rMBE (0.04% versus 0.03%) and the rRMSE (8.02% versus 6.90%).

model estimated the volume of the stem sections from the base to 9.00 m of trunk height with very good performance (Fig. 6).

Indeed, volume of stem sections computed integrating the Schumacher and Hall (1933) TLS-based model resulted very close to the volume computed using the traditional formulas, i.e. Huber, Smalian and cone, using the values of diameter and heights obtained from felled trees ($R^2 = 0.9728$, adjusted-$R^2 = 0.9727$, RMSE = 12.86 dm^3, rRMSE = 10.71% with df = 304, p-value < .0001).

Table 4 - The parameter estimates with the goodness-of-fit statistics for TLS-based Schumacher and Hall (1933) model calibrated with dimeter values extracted every 0.25 m.

	Schumacher and Hall (1933)			
Predictors	**Estimates**	**Standard error**	**Confidence interval**	**p-value**
a_0	2.1914	0.1674	1.8632-2.5196	<0.001
a_1	1.9325	0.0134	1.9062-1.9587	<0.001
a_2	1.3485	0.0156	1.3180-1.3791	<0.001
a_3	1.4814	0.0310	1.4207-1.5421	<0.001
Observations	34			
MBE	-0.0137 cm			
rMBE	-0.0413%			
RMSE	2.6576 cm			
rRMSE	8.0241%			
R^2	0.9536			

Comparison of volume estimates using traditional formulas and the most performing TLS-based model

Overall, the Schumacher and Hall (1933) TLS-based

Table 5 - Distribution of the errors (i.e. MBE, rMBE, RMSE and rRMSE) in the volume estimates according to the stem section by integrating the Schumacher and Hall (1933) TLS-based taper function with respect to the volume estimated with traditional formulas. Results from the application of Kolmogorov–Smirnov test are reported.

Stem section length range (m)	Distribution of the errors in volume estimates				Two-sample Kolmogorov-Smirnov test	
	MBE (dm³)	rMBE (%)	RMSE (dm³)	rRMSE (%)	D	p-value
0-1	-4.0375	-2.7146	14.6815	9.8709	0.0882	0.9996
1-2	-2.5661	-1.8193	10.0121	7.0985	0.0882	0.9996
2-3	2.7376	2.1125	8.5324	6.5841	0.1177	0.9762
3-4	3.4803	2.8349	11.2716	9.1815	0.1177	0.9762
4-5	2.8616	2.4381	12.3497	10.5220	0.08824	0.9996
5-6	1.0751	0.9496	14.7199	13.0016	0.08824	0.9994
6-7	0.6939	0.6441	19.1364	17.7634	0.08824	0.9994
7-8	2.7968	2.8014	10.4364	10.4536	0.08824	0.9996
8-9	-0.4309	-0.4426	11.5158	11.8292	0.08824	0.9996

The TLS-based model tends to underestimate the volumes in the lower and upper sections, whereas the mid-section volumes are overestimated. RMSE resulted lower than 15 dm^3 (i.e. rRMSE lower than 13%) in all stem sections except in log from 6 to 7 m (Tab. 5).

The Kolmogorov–Smirnov test indicates that the two distributions of volumes, those computed using the traditional formulas and those computed by the integration of the Schumacher and Hall (1933) TLS-based taper function, are not significantly different in all sections of the trunk.

Discussion

This study investigated the potentiality and the feasibility of terrestrial laser scanning measurements instead of traditional measurements, which entail the tree felling, to measure diameters along the stem, to develop taper function for European beech and estimate the volume of different stem sections.

Four equations calibrated with data collected by traditional field survey and TLS survey, that predict the diameter as a function of the height along the stem, were tested and their performance was assessed and compared. Between Schumacher and Hall (1933), Ormerod (1971), Yan (1992) and Kozak (2004) equations, the most performing was the Schumacher and Hall (1933) taper function, while the less performing was Yan (1992) both in case of traditionally collected and TLS measurements. The TLS-based taper function was more performant than the field-based taper equation, being the RMSE and the rRMSE of the first equation equal to 2.29 cm and 6.90% (versus 3.04 cm and 9.17%). Even according to the results of the Sun et al. (2016), the Schumacher and Hall (1933) TLS-based taper function was the most suitable equation but in their case with higher values of RMSE (7.82 cm versus 2.29 cm).

The model performance metrics of Schumacher and Hall (1933) TLS-based taper function did not increase when calibrated with TLS diameter and height data pairs extracted every 25.0 cm: indeed, the rRMSE raised from 6.90% to 8.02%. This outcome contradicted our hypothesis that the performance of the taper function would be improved if the equation would be built with diameter values extracted from the TLS point cloud at a higher frequency, i.e. 25.0 cm instead of 1.00 m. Because of this result, we can say that intensifying the computational efforts and consequently the time needed for data processing does not generate a model with higher predictive effectiveness.

The results from the analysis of the assessment of the error distribution of volume estimates along the stem computed integrating the TLS-based taper function showed that the sections with the highest bias are the first two sections at the base of the stem and the last one located at 8-9 m of height. The reason can be identified in the irregular shape of the lower part of the trunk that deviates from an almost regular cylinder. Indeed, the field measurement consists in callipering the section externally to buttresses, while the TLS procedure we used in this work to extract the diameter values fits an ellipse which tends to compensate diameter overestimations and underestimations along the section contour (see Fig. 4, lowest sections). Thus, diameter measures from calipering and TLS may differ even when the section area estimated by TLS is accurate. This aspect is another strength of the TLS approach that can compensate for biasing stem irregularities. The bias in the last section can be attributable to the occlusions caused by leaves and branches during TLS scans. Indeed, the higher the section is, the higher the probability for the occlusions within the single-scan TLS data. Nonetheless, the volume estimates in the different sections of the stem by integrating the TLS-based taper function resulted not significantly different from the volume estimates using the sectional methods based on traditional formulas. This means that the two approaches, traditional and TLS surveying methods, lead to the same volume estimates, but TLS does not require the tree felling. In general, the analyses conducted in this study suggest that TLS is a suitable approach for developing stem taper function without the need for destructive sampling.

Referring to methodological aspects, we can state that methods of scanning and fitting on tree point clouds are quite different and often customized (Pueschel et al 2013, Puletti et al 2019). The performance of taper equations built with TLS data acquired from single and multi-scans in forest plots (Liang et al. 2018) should be tested, as well as the effect of vertical slice thickness of the point cloud sample and the procedure to fit the stem and to extract the diameter (ellipses versus circle). Nevertheless, we know from the Saarinen et al. (2017) study, aimed at investigating how TLS data for deriving stem volume information from single trees should be collected, that when two TLS point clouds were utilized instead of one the accuracy of stem volume estimates improved. Our study confirmed this result also for European beech trees - of mount Amiata - that present generally a more irregular stem shape than Scots pine and Norway spruce and birch (*Betula* spp. L.). This aspect is a further strength of the TLS non-destructive method. Regarding the sample size, its effect on the performance of taper equations have been investigated by Subedi et al. (2011): the outcomes of their study show that the minimum number of trees required to model taper equations without compromising model accuracy depends on tree species and the model form used to describe tree taper. Specifically, through a stratified random sampling, the minimum number of trees required was 15 for jack pine and 30 for black spruce for the variable-exponent taper equation developed by Sharma and Zhang (2004). For the modified variable-exponent taper equation developed by Kozak (2004), however, the minimum number of trees required were 80 and 50 for jack pine and black spruce, respectively. Furthermore, we realized leaf-on scanning which is the worst phenological condition for TLS survey. Using leaf-off scanning in beech trees, i.e. from October to February in mount Amiata, could probably lead to even better results.

Conclusions

The investigation presented here has the value of a case study and represents the first attempts to develop a taper equation for the European beech of mount Amiata. Further verifications should be done to build a taper equation valid for the entire region.

The results demonstrate that the statistics of stem taper models developed using TLS data are better than those of models developed with data collected with a traditional destructive method. This is a preliminary study; future investigations will be carried out to confirm that TLS provides accurate and detailed non-destructive measurements suitable to develop taper function for European beech in case of different approaches of TLS data acquisition and different sample sizes.

Scanning trees of a forest stand at multiannual intervals, that means to scan only a portion of trees for the occasion, allows to quantify the evolutionary change of the stand and through the taper equations the evolution of stem profile is depicted in detail. In this perspective, the non-destructive TLS derived data allow updating taper equation according to the forest growth obtaining flexible and adaptable models for every development stage, allowing for the development of very detailed yield models. In the timber industry, the detailed knowledge of the available quantity of volume and assortments from a trunk is important for waste minimization and consequently for supply chain efficiency optimization.

Considering the high levels of accuracy in tree measurements introduced by TLS, which can enhance the quality of the information on the quantity of the extractable volume from a trunk, the diffusion of taper equations is desirable generating benefits in the wood chain supply and the flow of raw materials. Besides, a further advantage of the stem taper equation, compared to the classical volume tables with assortments information (when eventually available), is that in case of changes of dimensional standards required by the market, the function is still efficient, unlike the tables necessarily based on fixed thresholds. Moreover, the use of taper equation shifts the attention from generic wood volume available for wood supply to timber assortments helping to enforce the “cascading use of wood” objective. In Italy the availability of taper equations is really limited, the diffusion of this approach assisted by TLS non-destructive survey can strongly and positively contribute to the goal of the sustainable forestry.

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